Written Calculation Policy 2022-24

Updated: Spring 2022

Review: Autumn 2024

woodstock
CE PRIMARY SCHOOL

Nurture ~ Believe ~ Discover ~ Achieve

OUR CHRISTIAN VISION

Our vision for Woodstock CE Primary School reflects a passionate commitment to learning and recognition of the uniqueness of individual learners. Guided by our Christian values, it is driven by our desire to offer the best possible education for our pupils in partnership with parents, the Church and the local community.

Woodstock CE Primary School will be a centre for learning where adults and children:

- ✓ ***Nurture*** and prioritise wellbeing and development.
- ✓ ***Believe*** in themselves and in each other.
- ✓ ***Discover*** their own strengths and become successful lifelong learners.
- ✓ ***Achieve*** more than they ever thought possible.

OUR CHRISTIAN ETHOS

Recognising our historic foundation, we will preserve and develop our religious character in accordance with the principles of the Church of England and in partnership with the Churches at parish and diocesan level.

Woodstock CE Primary School strives to be an inclusive community where children grow, learn and achieve together. Within a nurturing, supportive and safe environment, mental health and wellbeing is at the heart of everything we do and recognised as the responsibility of all. Children's natural curiosity is fostered through a creative curriculum that excites and challenges, and enables them to be successful learners. Supported by a culture of equality and aspiration we aim to remove disadvantage so that every child can thrive.

We are committed to providing an education of the highest quality within the context of Christian belief and practice. We encourage an understanding of the meaning and significance of faith, and promote Christian values through the experience we offer to all our pupils.

"For I know the plans I have for you", declares the Lord, ***"plans to prosper you and not to harm you, plans to give you hope and a future."*** Jeremiah 29, v11

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Introduction

As a school we recognise the important link between mental, practical and written methods to support the development of children's understanding. Children are introduced to the processes of calculation through practical, oral and mental activities. As they begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, so that they develop both conceptual understanding and fluency in the fundamentals of mathematics. Whilst interpreting signs and symbols involved with calculation, orally in the first instance, children use models and images to support their mental and written methods of calculation. As children's mental methods are strengthened and refined they begin to work more efficiently, which will support them with using succinct written calculation strategies as they are developed.

By the time children leave Woodstock CE Primary School they will be equipped with efficient mental and written calculation methods, which they use with fluency. Decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. At whatever stage in their learning, and whatever method is being used, children's strategies must still be underpinned by a secure understanding and knowledge of number facts that can be recalled fluently.

The overall aims are that when children leave Woodstock Primary School they:

- Are able to recall number facts with fluency, having developed conceptual understanding through being able to visualise key ideas – such as those related to place value - through experience with practical equipment and visual representation.
- Make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads.
- Have an efficient, reliable, written method of calculation for each number operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally.
- Are able to make connections between all four number operations, understanding how they relate to one another, as well as how the rules and laws of arithmetic can be applied.

Mental Methods of Calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. On-going oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined in order to develop fluency. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills.

Written Methods of Calculation

In line with the National Curriculum 2014, our emphasis is on ensuring that pupils progress quickly towards efficient methods. This guidance promotes the use of what are commonly known as 'formal' written methods – methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have a reliable and efficient written method to apply to calculations.

In setting out these aims, the intention is that we adopt greater consistency in our approach to calculation. The challenge is for our teachers to determine when their children should move on to a refinement in the method and know when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

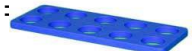
We value the communication between teachers and pupils and pupils and their peers. When children feel confident to communicate their ideas and discuss their findings openly this improves their level of understanding. It has been proved that children will remember 70% of what they have been learning if they have taken an active part in the lesson, compared to a passive learner who will only retain 20% of what has been taught.

Choosing the Appropriate Strategy

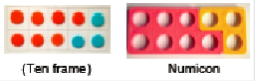
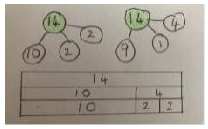
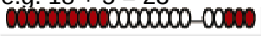
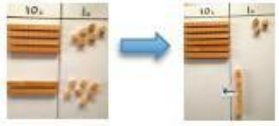
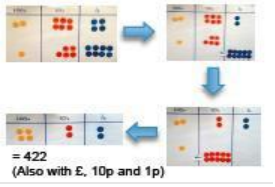
Recording in mathematics, and in calculation in particular is an important tool both for furthering the understanding of ideas and for communicating those ideas to others. An efficient written method is one that helps children carry out a calculation and can be understood by others. Written methods are complementary to mental methods and should not be seen as separate from them. The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. It is important children acquire secure mental methods of calculation and fluency in using and applying efficient written methods of calculation for addition, subtraction, multiplication and division.

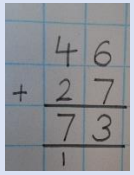
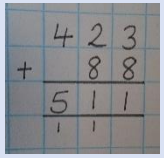
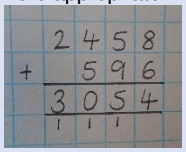
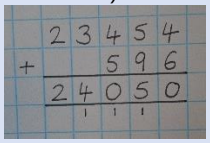
First Experiences/Building Blocks:

There are fundamental skills that it is important for children to develop the building blocks to future learning in maths, including that linked to calculation. These will be taught initially in EYFS/Year 1 but also revisited throughout KS1 and KS2 to ensure depth of understanding. These skills include:

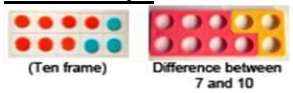
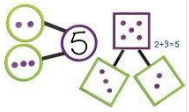
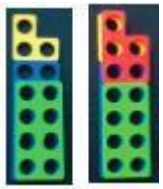
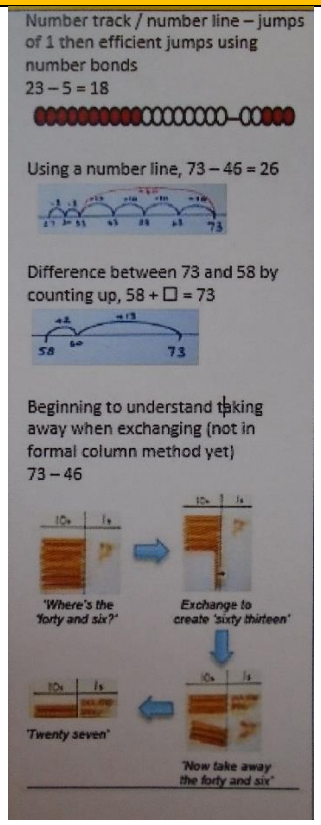
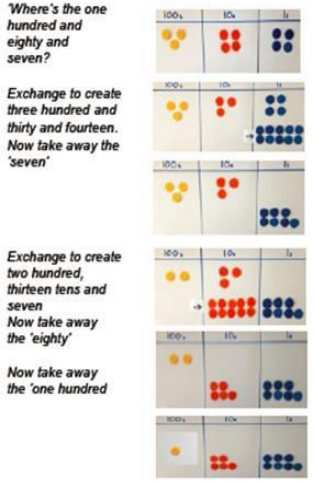
- **Ordinality** – 'the ordering of numbers in relation to one another' – e.g. (1, 2, 3, 4, 5…)
- **Cardinality** – 'understanding the value of different numbers' – e.g. (7 =  17   12 = 
- **Equality** – 'seven is the same value as four add three' – e.g  = 
- **Subitising** – 'instantly recognizing the number of objects in a small group, without counting them'
  e.g.  → five
- **Conservation of number** – 'recognising that a value of objects are the same, even if they are laid out differently' e.g.
  
- **Counting on and back from any number**
  e.g. 'five add three more totals eight'  , 'ten take away three totals seven' 
- **Using apparatus and objects to represent and communicate thinking**
  e.g. 
- **Maths language** – using mathematical words verbally in every-day situations
  e.g. 'climb up to the top' / 'climb down to the bottom'

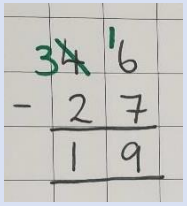
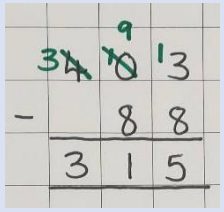
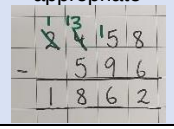
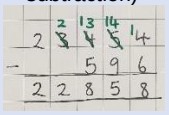
ADDITION

	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Calculation expectations to be solved using a range of strategies. See below:	Knowing the numbers 1 – 20 and knowing what is one more than any number to 20.	To add one digit and two-digit numbers to 20. To solve additions using missing number problems e.g. 7 + ☐ =9	Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers • adding three one-digit numbers	Add and subtract numbers mentally, including: • a three-digit number and ones • a three-digit number and tens • a three-digit number and hundreds • Add up to 3 digits using compact column method	Add and subtract numbers with up to 4 digits using compact column method where appropriate.	Add and subtract whole numbers with more than 4 digits using compact methods. Add and subtract numbers mentally with increasingly large numbers.	Perform mental calculations, including with mixed operations and large numbers
Developing conceptual understanding	Partition numbers to 10 in different ways.   Count on from the biggest number using resources.  Count on, on a number track, in 1s 	Number bonds to 20   Use bonds of 10 to calculate bonds of 20  Count on  Count on, on number track, in 1s 	Number track / number line – jumps of 1 then efficient jumps using number bonds e.g. 18 + 5 = 23  e.g. 46 + 27 = 73 Count in tens then bridge  e.g. 25+ 29 by +30 then -1 (round and adjust)  Partition and recombine 46 + 27 = 60 + 13 = 73 	Use known facts to derive new ones e.g. 40 + 80 = 120, using 4 + 8 = 12, so 400 + 800 = 1200 Round and adjust e.g. 243 + 198 by +200 then -2  Pairs that make 100 23 + 77  Calculating using place value counters (100s, 10s, 1s) to understand compact methods. e.g. 264 + 158 	When starting new domains use Numicon, coins, Cuisenaire and double-sided counters to aid conceptual understanding.	When starting new domains use Numicon, coins, Cuisenaire and double-sided counters to aid conceptual understanding. Place value counters to be used to develop place value understanding.	When starting new domains use Numicon, coins, Cuisenaire and double-sided counters to aid conceptual understanding. Place value counters to be used to develop place value understanding.

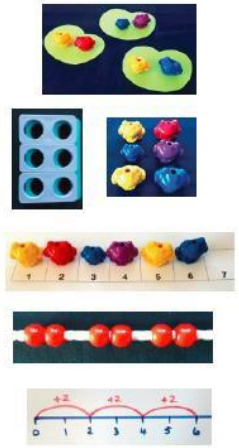
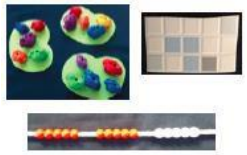
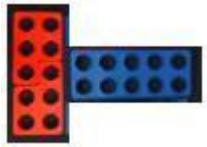
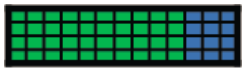
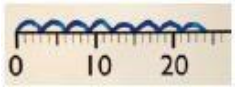
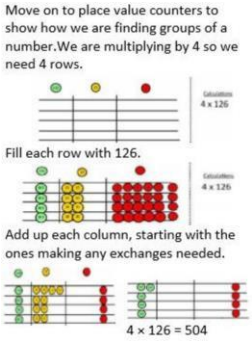
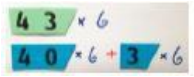
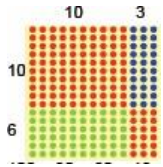
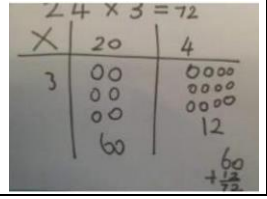
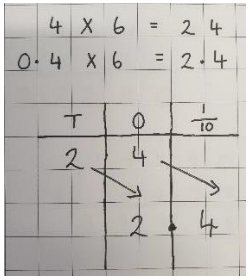
Mental calculation strategies	1:1 correspondence	1 more and	10 more Number bonds: 20, 12, 13	Add multiples of 10, 100	Add multiples of 10s, 100s and 1000s	Add multiples of 10s, 100s, 1000s,10000,100000 and 1000000 and tenths	Add multiples of 10s, 100s, 1000s, 10000,100000 and 1000000, tenths and hundredths
	Concept of zero	2 more	Number bonds: 14, 15 Add a one-digit number to two-digit number by bridging	Add single digit bridging through boundaries	Fluency of 2-digit + 2-digit not relying on compact methods	Fluency of 2 digit + 2 digit including with decimals	Fluency of 2 digit + 2 digit Including with decimals
	Conservation of number (quantity remains the same regardless of organisation)	10 more	Partition second number, add tens then ones	Partition second number to add pairs to 100	Partition second number to add Decimal pairs of 10 and 1	Partition second number to add	Partition second number to add
	1 more	Largest number first.	Add 10 and multiples Number bonds: 16, 17	Use near doubles to add	Use near doubles to add	Use number facts, bridging and place value	Use number facts, bridging and place value
	Largest number first	Doubles up to 12	Doubles up to 20 and multiples of 5 Add near multiples of 10	Add near multiples of 10 and 100 by rounding and adjusting	Adjust both numbers before adding Add near multiples	Adjust numbers to add	Adjust numbers to add
	Counting on	Near doubles to 10	Number bonds: 18, 19 Partition and recombine	Partition and recombine	Partition and recombine	Partition and recombine	Partition and recombine
	Using number bonds	Using number bonds	Using known facts	Using known facts	Using known facts	Using known facts	Using known facts
Just know it!	Children should know their number bonds for all numbers up to 10	Children should know all their number bonds to 20 and doubles to 10	Recall and use addition and subtraction facts to 20 fluently and derive/use related facts up to 100	Derive new facts using bonds learnt in KS1	Derive new facts using bonds learnt in KS1	Derive new facts using bonds learnt in KS1	Derive new facts using bonds learnt in KS1
Written Methods	**Using objects and quantities, children add and subtract using two single digit numbers.** Children to begin to mark make to express calculations Formal number sentences not to be taught. <>= to be used to compare numbers	**Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equal (=) signs** Children to write addition calculations in standard and non-standard ways. For example: ☐ 4 + 2 3 + ☐ + 7 = 14	**Add and subtract two two-digit numbers** using concrete objects, pictorial representations progressing to formal written methods 	**Add and subtract numbers with up to three digits**, using formal written methods of columnar addition and subtraction 	**Add and subtract numbers with up to four digits**, using formal written methods of columnar addition where appropriate 	**Add and subtract whole numbers with more than 4 digits**, including using formal written methods (compact addition and subtraction) 	Solve addition and subtraction **multi-step problems** in contexts, **deciding which methods to solve them and why (i.e. compact, mental or calculator)**

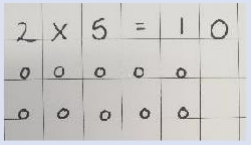
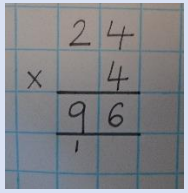
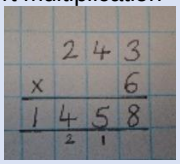
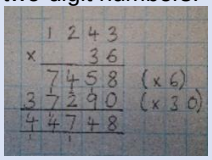
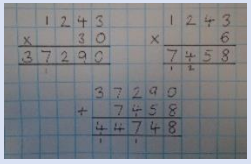
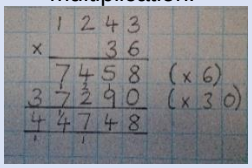
SUBTRACTION

	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Calculation expectations to be solved using a range of strategies. See below:	Knowing the numbers 1 – 20 and knowing what is one less/fewer than any number to 20.	To subtract one digit and two digit numbers to 20. To solve subtraction problems using number problems such as 7 = ☐ - 9 Use number bonds for subtraction facts within 20.	Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers	Add and subtract numbers mentally, including: • a three-digit number and ones • a three-digit number and tens • a three-digit number and hundreds • subtract up to 3 digits using compact column method	Add and subtract numbers with up to 4 digits using compact column method where appropriate.	Add and subtract whole numbers with more than 4 digits using compact methods. Add and subtract numbers mentally with increasingly large numbers.	Perform mental calculations, including with mixed operations and large numbers
Developing conceptual understanding	**Partition numbers to 10 in different ways.**   **Count back to subtract using resources.**  7 - 4 = 3 (children do not need to record number sentences) **Count back on a number track, in 1s** 	**Number bonds to 20** **Count back** Count back, on a number track. 15 – 6 = 9  Difference between 13 and 8 13 – 8 = ☐ 8 + ☐ = 13 	 	**Taking away and exchanging** 344 – 187  	When starting new domains use Numicon, coins, Cuisenaire and double-sided counters to aid conceptual understanding	When starting new domains use Numicon, coins, Cuisenaire and double-sided counters to aid conceptual understanding Place value counters to be used to develop place value understanding	When starting new domains use Numicon, coins, Cuisenaire and double-sided counters to aid conceptual understanding Place value counters to be used to develop place value understanding

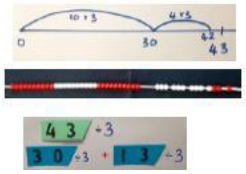
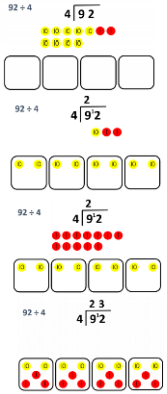
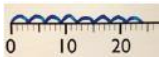
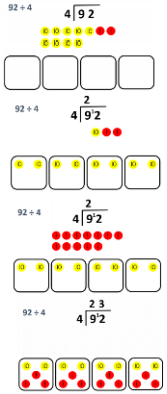
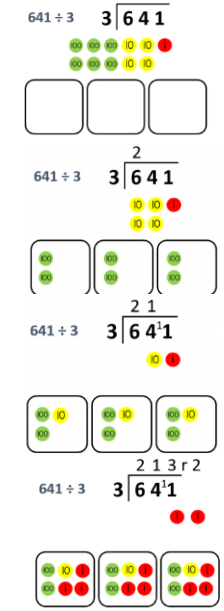
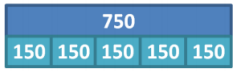
Mental calculation strategies	1:1 correspondence	1 less/fewer	10 less	Subtract multiples of 10, 100	Subtract multiples of 10s, 100s and 1000s	As before and 10,000 100,000 and 1,000,000 and tenths	As before and hundredths
	Concept of zero	2 less/fewer	Subtract 1 digit from 2 digit by bridging	Subtract single digit bridging through boundaries	Fluency of 2-digit subtract 2-digit	Fluency of 2 digit - 2 digit including with decimals	Fluency of 2 digit - 2 digit Including with decimals
	Conservation of number (quantity remains the same regardless of organisation)	10 less	Partition second number, count back in 10s then 1s	Partition second number to subtract	Partition second number to subtract Decimal subtraction from 10 or 1	Partition second number to subtract	Partition second number to subtract
	1 less/fewer	You subtract from the whole	Subtract 10 and multiples of 10			Use number facts, bridging and place value	Use number facts, bridging and place value
	Number bonds, subtraction: 1, 2	Halves up to 20	Halves up to 100 (multiples of 10)	Subtract near multiples of 10 and 100 by rounding and adjusting	Subtract near multiples by rounding and adjusting	Adjust numbers to subtract	Adjust numbers to subtract
	Count back Number bonds, subtraction: 3, 4	Difference between	Difference between	Difference between	Difference between		
	Using number bonds	Using number bonds	Subtract near multiples of 10 11,19,21,29	Subtract multiples of 10, 100	Subtract multiples of 10s, 100s and 1000s	Using known facts	Using known facts
Just know it!	Children should know their number bonds for all numbers up to 10	Children should know all their subtraction number bonds to 20 and halves to 20	Recall and use addition and subtraction facts to 20 fluently and derive/use related facts up to 100	Derive new facts using bonds learnt in KS1	Derive new facts using bonds learnt in KS1	Derive new facts using bonds learnt in KS1	Derive new facts using bonds learnt in KS1
Written Methods	**Using objects and quantities, children add and subtract using two single digit numbers.** Children to begin to mark make to express calculations Formal number sentences not to be taught. <>= to be used to compare numbers	**Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equal (=) signs** Children to write addition calculations in standard and non-standard ways. For example: ? = 4 – 2 13 - ? = 7	**Add and subtract two two-digit numbers** using concrete objects, pictorial representations progressing to formal written methods 	**Add and subtract numbers with up to three digits**, using formal written methods of columnar addition and subtraction 	**Add and subtract numbers with up to four digits**, using formal written methods of columnar addition where appropriate 	**Add and subtract whole numbers with more than 4 digits**, including using formal written methods (compact addition and subtraction) 	Solve addition and subtraction **multi-step problems** in contexts, **deciding which methods to solve them and why (i.e. compact, mental or calculator)**

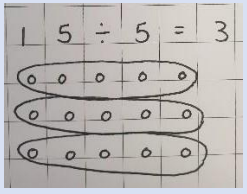
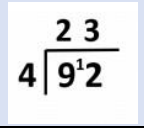
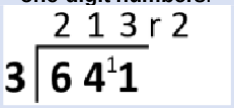
MULTIPLICATION

	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Calculation expectations to be solved using a range of strategies.	They solve problems doubling, halving and sharing	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	Write & calculate mathematical statements for x and ÷ using the multiplication tables that they know, (inc. two-digit numbers x one-digit numbers), using mental & progressing to formal written methods Solve problems, including missing number problems, involving multiplication	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication to the context
Developing conceptual understanding	Equal grouping 	Equal grouping 	Equal grouping Use of multiplication facts e.g. 5 x 3 = 15  Arrays to expose the commutative law e.g. 5 x 2 = 2 x 5  Build tables on counting stick  Link to repeated addition 	The distributive law (partitioning) 13 x 4 = (10 x 4) + (3x4)   Continue to build tables using counting stick and other relevant resources    Developing conceptual understanding of the written method through use of concrete manipulatives  Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows. Fill each row with 126. Add up each column, starting with the ones making any exchanges needed. 4 × 126 = 504	The distributive law (partitioning) 43 x 6 40 x 6 = 240 3 x 6 = 18 43 x 6 = 258   Deriving other known facts If I know 4 x 6 = 24 Then 4 x 60 is ten times bigger Continue to build tables using counting stick and other relevant resources Continue to develop conceptual understanding of the written method through use of concrete manipulatives 	Deriving other known facts including relating to decimals e.g. If I know 4 x 6 then 0.4 x 6 is ten times smaller. 	

Mental calculation strategies	Grouping	Count in 2s	Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables	Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	Recall multiplication and division facts for multiplication tables up to 12 × 12	Identify multiples using known number facts	Multiply numbers mentally drawing upon known facts
	Doubling (using numbers up to 10)	Count in 10s	Show that multiplication of two numbers can be done in any order (commutative)	Doubling	Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1	Multiply numbers mentally drawing upon known facts	Doubling and near doubles
		Count in 5s	Recognising odd and even numbers	Near doubles	Recognise and use factor pairs and commutativity in mental calculations	Multiply whole numbers and those involving decimals by 10, 100 and 1000 using PV knowledge	
		Doubling (using numbers up to 10)			Multiplying together three one-digit numbers, making choices about which order to do them in	Doubling and near doubles	
		Double multiples of 10			Doubling and near doubles		
Just know it!		Doubling (using numbers up to 10)	Multiplication and division facts for 2,5 and 10	Multiplication and division facts for 2,5 and 10 Multiplication and division facts for 3, 4 and 8	Multiplication and division facts for all times tables	Multiplication and division facts for all times tables Recall of square and cube numbers, initially drawing on times tables knowledge.	Multiplication and division facts for all times tables
Written Methods	**Solve problems using doubling.** Children to begin to mark make to express calculations Formal number sentences not to be taught (SYMBOLS NOT TO BE INTRODUCED)	**Solve problems using doubling.** Children begin to understand multiplication and division through grouping and sharing small quantities. They solve one step problems by calculating the answer using concrete objects, pictorial representations and arrays.	**Calculate mathematical statements for multiplication within the multiplication tables** (2,5&10) and write them using the multiplication (x) and equals (=) signs. 	Write and calculate mathematical statements for x using the x tables they know (2,5,10,3,4&8), progressing to **formal short multiplication with two-digit numbers multiplied by one-digit numbers**. 	**Multiply up to a three-digit numbers by a one-digit number** using formal short multiplication 	**Multiply numbers up to 4 digits by one- or two-digit number** using a formal written method, **including long multiplication** for two-digit numbers.  Or decomposing in to 2 separate calculations 	**Multiply multi-digit numbers up to 4 digits by a two-digit whole number** using the formal written method of long multiplication. 

DIVISION

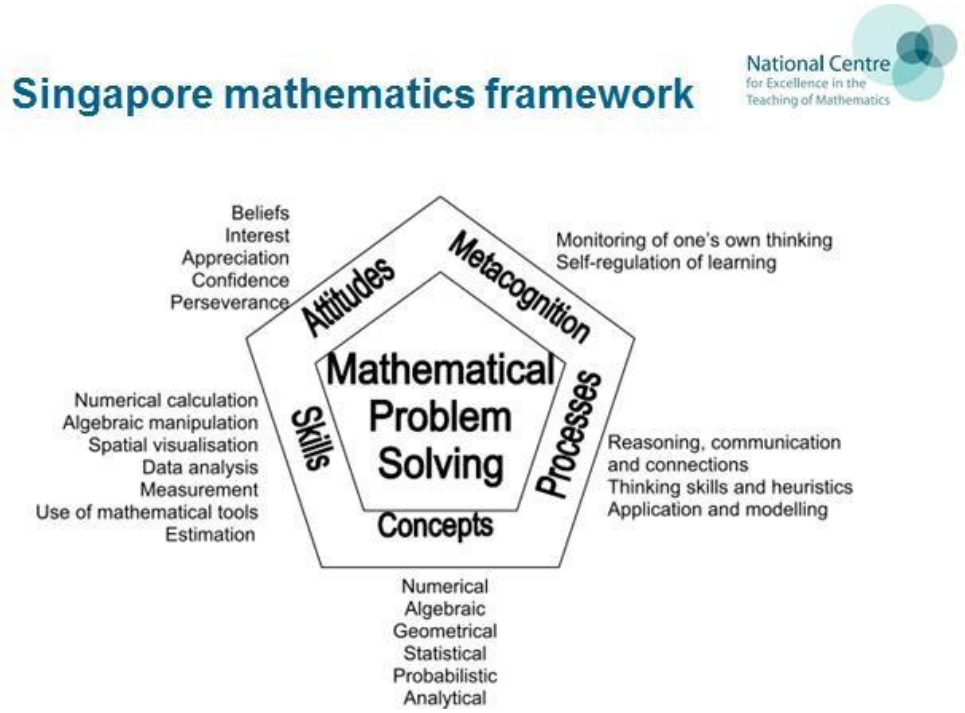
	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Calculation expectations to be solved using a range of strategies.	They solve problems doubling, halving and sharing	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	Write and calculate mathematical statements for x and ÷ using the multiplication tables that they know, (inc. for two-digit numbers times one-digit numbers), using mental and progressing to formal written methods Solve problems, including missing number problems, involving multiplication and division	Solve problems involving multiplying and division	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication to the context
Developing conceptual understanding	<u>Equal sharing</u> 6 ÷ 2 = 3 by sharing into 2 groups 	<u>Equal sharing</u> 6 ÷ 2 = 3 by sharing into 2 groups 6 ÷ 2 = 3 by sharing into 2 groups and by grabbing groups of 2  <u>Grouping</u> How many groups of 2 make 6? 	<u>Equal sharing</u> 15 ÷ 3 = 5 in each group (sharing)  <u>Grouping linked to fractions</u> Splitting in to 3 equal groups is the same as finding thirds.  <u>Grouping</u> 15 ÷ 3 = 5 groups of 3 (grouping)  How many groups of 2 make 6?  <u>Use language of division</u> linked to tables 10 ÷ 2 = 5 	<u>Mental strategies using partitioning and known facts</u> 43 ÷ 3, if I know 10 x 3…  <u>Use language of division</u> linked to tables How many groups of 3 ~~make 24?~~  <u>Using place value counters to support written method</u> 132 ÷ 3 	<u>Grouping using known facts for partitioning</u> 196 ÷ 6 If I know 3 x 6 … then 30 x 6  <u>Use language of division</u> linked to tables How many groups of 3 ~~make 24?~~  <u>Using place value counters to support written method</u> 132 ÷ 3  <u>Short division</u> 641 ÷ 3 = 213 r2 2 1 3 r 2 3 ⟌ 6 4¹1	<u>Using place value counters to support written method</u>  <u>Bar model used to reinforce 'how many [divisors] in [dividend]?'</u> 750 ÷ 150  750 150 150 150 150 150	<u>Using place value counters to support written method</u> Extend previous method to include exchanging ones for tenths to model decimal remainders

Mental calculation strategies	Sharing equally	Sharing equally	Sharing equally	Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables	Recall and use multiplication and division facts for multiplication tables up to 12 × 12	Recall and use multiplication and division facts for multiplication tables up to 12 × 12	Divide numbers mentally drawing upon known facts
	Halving equally	Halving equally	Halving equally	Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1	Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1	Use knowledge of factors and multiples to determine whether a number will have a remainder
		Counting in 2s, 5s & 10s	Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables	Halving equally	Recognise and use factor pairs in mental calculations (e.g. I know that 3 and 12 are factors of 36, therefore I know that 36 ÷ 12 = 3)	Recognise and use factor pairs in mental calculations (e.g. I know that 3 and 12 are factors of 36, therefore I know that 36 ÷ 12 = 3)	
		Use counting for grouping	Use multiplication facts for grouping	Recall half of any even number up to 100	Strategies for finding simple unit fractions of a number	Strategies for finding simple unit fractions of a number	
			Recognising odd and even numbers (this leads in to knowing whether a number is divisible by 2)	Recall half of any multiple of ten		Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	
			Half of any even number up to 100				
Just know it!			Half of any multiple of 10	Multiplication and division facts for 2,5 and 10	Multiplication and division facts for 3, 4 and 8	Multiplication and division facts for all times tables	Multiplication and division facts for all times tables
			Half of any even number up to 20			Recall prime numbers up to 19	Recall prime numbers up to 19
Written Methods	**Solve problems using halving and sharing.** Children to begin to mark make to express calculations Formal number sentences not to be taught (SYMBOLS NOT TO BE INTRODUCED)	**Solve problems using halving and sharing.** Children begin to understand division through grouping and sharing small quantities. They solve one step problems by calculating the answer using concrete objects, pictorial representations and arrays.	**Calculate mathematical statements for division within the multiplication tables** (2,5&10) and write them using the division (÷) and equals (=) signs. 	Write and calculate mathematical statements for ÷ using the x tables they know (2,5,10,3,4&8), progressing to **formal short multiplication with two-digit numbers divided by one-digit numbers**. 	Write and calculate mathematical statements for ÷ using the x tables knowledge using **formal short multiplication with three-digit numbers divided by one-digit numbers**. 	Divide numbers up to 4 digits by a one-digit number using the **formal written method of short division** and interpret remainders appropriately for the context See previous for example of methods	Divide numbers up to 4 digits by a two-digit number using the **formal written method of short division** where appropriate, interpreting remainders according to the context See previous for example of methods

Appendix 1:

Progression in Use of the Bar Model – Written by the Oxfordshire Maths Team – July 2014

The bar model was introduced in Singapore in the 1980s alongside an increased emphasis on problem solving in the curriculum. A new curriculum was devised based heavily on the Cockcroft report (1982) and 'Agenda in Action' (1989) – with the latter being an American report. It fits a maths mastery model, which involves spending more time on a topic. This allows students opportunities to explore maths in-depth. Such a concept has also been stressed by Jane Jones; lead HMI for mathematics, when she talks about 'milking the maths.'

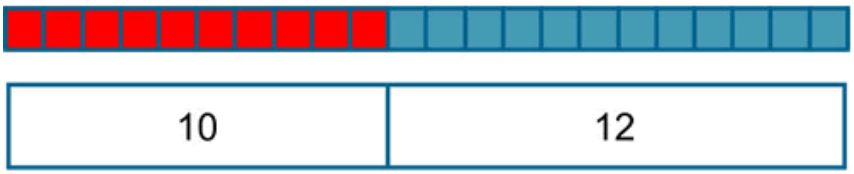


The purpose of this document is to illustrate progression within the bar model and is the Oxfordshire maths team's attempt to show where it could fit within the new national curriculum. It could become part of a school's calculation policy. Please note that use of the bar model is not statutory and is designed to help children visualise maths problems, rather than do the maths for them. We have drawn heavily on various NCETM resources in order to write this document.

The Singapore and Shanghai curriculums have problem solving at the heart of them. Arguably, this is why they are so successful. This model also reflects strong teaching in this country, as noted in OfSTED's 'Made to Measure' report. (2012)

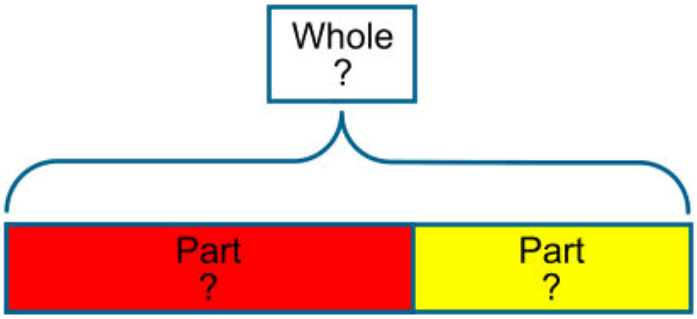
What does the bar model look like?

Sam had 10 red marbles and 12 blue marbles. How many marbles did he have altogether?

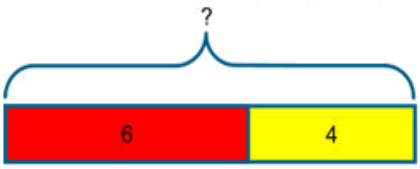


10 + 12 = 22

In problems involving addition and subtraction there are three possible unknowns as illustrated below and given the value of two of them the third can be found.



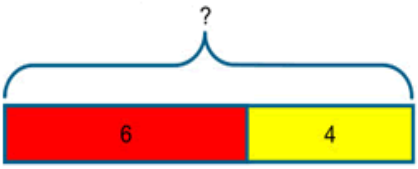
Addition
Aggregation
- two quantities combined



I have 6 red pencils and 4 yellow pencils. How many pencils do I have?

(I combine two quantities to form the whole)

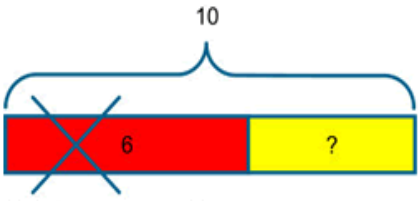
Addition
Augmentation
- a quantity is increased



I have 6 red pencils and I buy 4 yellow pencils. How many pencils do I have?

(The bar I started with increases in length)

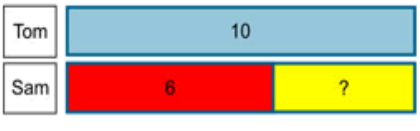
Subtraction
- Take Away



I had 10 pencils and I gave 6 away, how many do I have now?

(This time we know the whole but only one of the parts, so the whole is partitioned and one of the parts removed to identify the missing part)

Subtraction
- Comparison or Difference



Tom has 10 pencils and Sam has 6 pencils. How many more does Tom have?

(The bar is particularly valuable for seeing the difference between the two quantities)

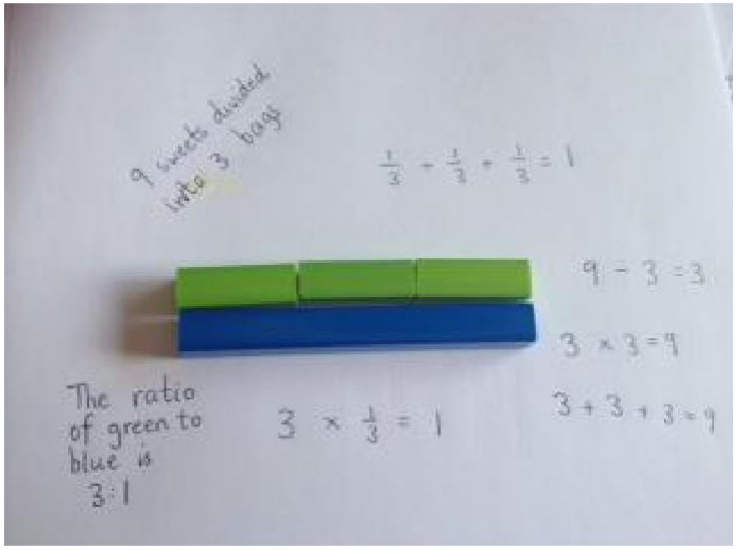
Equivalence

The model can be rearranged to demonstrate equivalence in a traditional layout

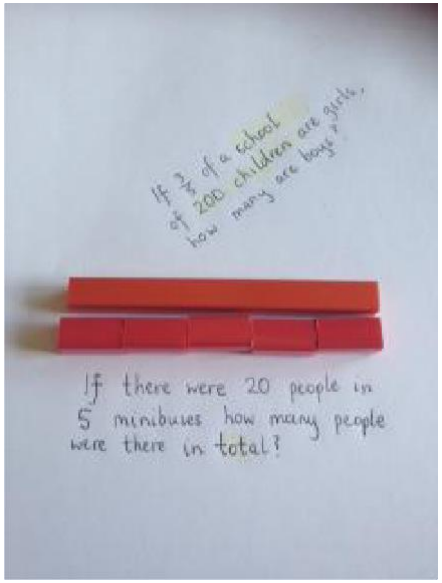
Pupils need to develop fluency in using this structure to represent addition and subtraction problems in a variety of contexts using the bar model. The model will help children to see that different problems share the same mathematical structure and can be visualised in the same way. Asking children to write their own problems, using the bar as the structure will help to consolidate this understanding.

Milking the maths: using the bar model flexibly across all year groups

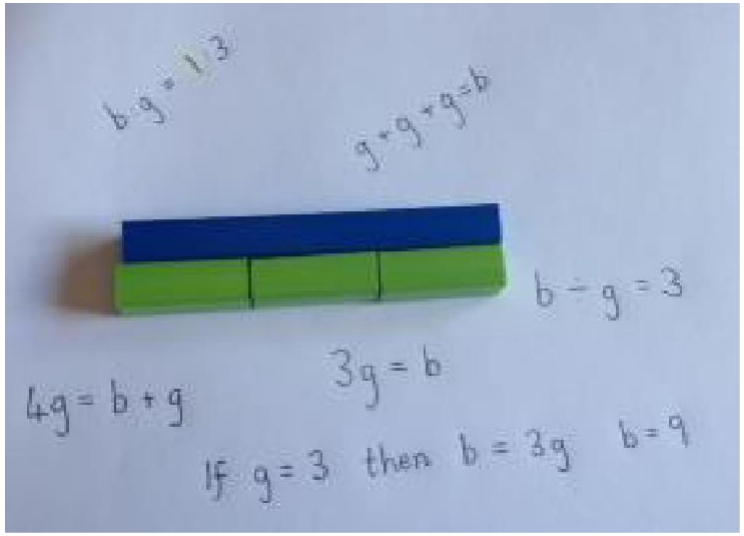
What could this bar model be showing?



Can you design questions that fit the bar model?



Playing with algebra



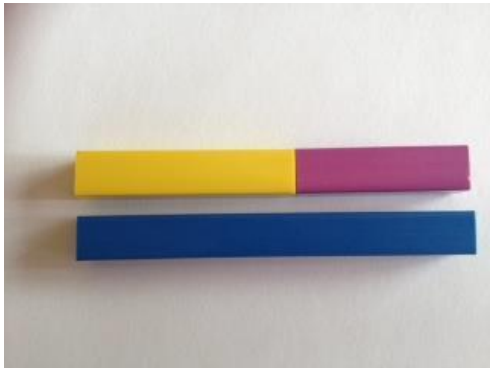
We feel that Cuisenaire should be used by children at this stage, so that the gap is bridged between using concrete objects and drawing symbolic representations of objects. We would expect children to be working practically with Cuisenaire and talking about relations, so that they are then ready to start drawing bar model representations in lower Key-Stage 2.

National Curriculum Objectives: Rapid Recall

Year 1: represent and use number bonds and related subtraction facts within 20
Year 2: recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100

National Curriculum Objectives: Addition and Subtraction

Year 2: show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot



Focus on verbalising thinking:
e.g.
'five add four is equal to nine'
'four add five is equal to nine'
'nine take-away four equals five'
'nine take away five equals four'

Addition and Subtraction

The bar model supports understanding of the relationship between addition and subtraction in that both can be seen within the one representation and viewed as different ways of looking at the same relationships.

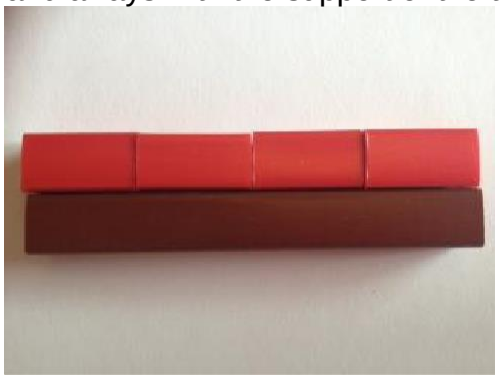
This diagram encapsulates all of the following relationships;

$a = b + c$; $a + c + b$; $a - b = c$; $a - c = b$

For teacher's understanding, children would not be expected to write the above algebraic notations.

National Curriculum Objectives: Multiplication and Division

Year 1: solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representati ons and arrays with the support of the teacher.



Focus on verbalising thinking:
e.g.
'eight is two taken four times'
'two taken four times is eight'
'eight equals four times two'
'there are four twos in eight'

Year 2: solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
[Photograph of Cuisenaire being used as a bar model and arrays.]

National Curriculum Objectives: Fractions

Year 1: recognise, find and name a half as one of two equal parts of an object, shape or quantity
recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.
Year 2: recognise, find, name and write fractions $^{1}/_{3}$, $^{1}/_{4}$, $^{2}/_{4}$ and $^{3}/_{4}$ of a length, shape, set of objects or quantity
write simple fractions e.g. $^{1}/_{2}$ of 6 = 3 and recognise the equivalence of $^{2}/_{4}$ and $^{1}/_{2}$.

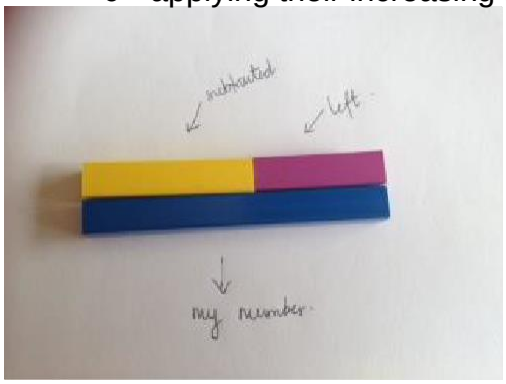
e.g.
What fractions can you see?
What fraction of the orange is each yellow piece?
If the value of the orange rod is ten, what is the value of each yellow rod?
Extension: What if the value of the orange rod is 6? 100? Etc.

National Curriculum Objectives: Problem Solving

Year 1: solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9.

Year 2: solve problems with addition and subtraction:

- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods

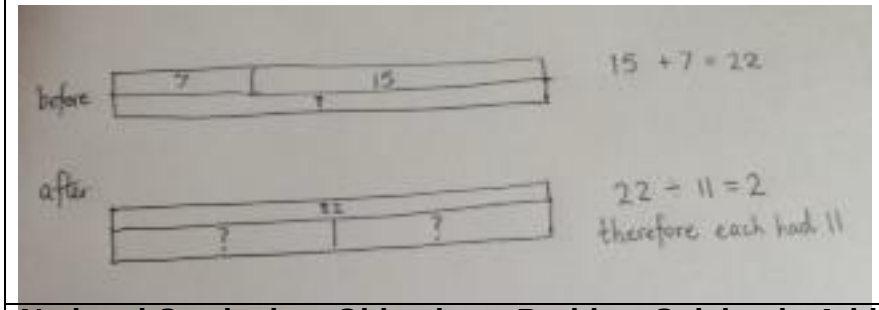


e.g.
I think of a number. I subtract 5. The answer is 4. What is my number?

e.g.
A tub contains 24 coins. Saj takes 5 coins. Joss takes 10 coins. How many coins are left in the tub?

Years 3 and 4:

We feel that children should now be ready to start drawing bar model representations. For the purpose of this progression document, we have modelled 'before' and 'after' representations of problems.

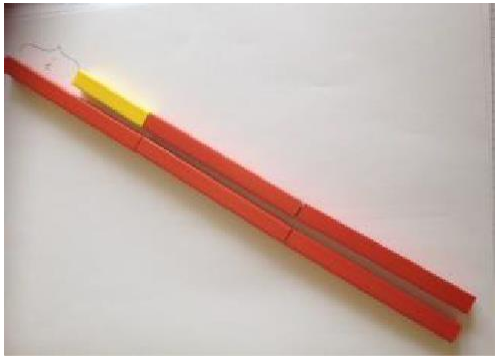


e.g.

Aiden has seven marbles and Harvey has fifteen. They decide to share them equally between them. How many do they get each?

National Curriculum Objectives: Problem Solving in Addition and Subtraction

Year 3: solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.
Year 4: solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.



e.g. one-step problem
Sally has 40 football cards. She gives 25 of them away. How many does she give away?

National Curriculum Objectives: Problem Solving in Multiplication and Division

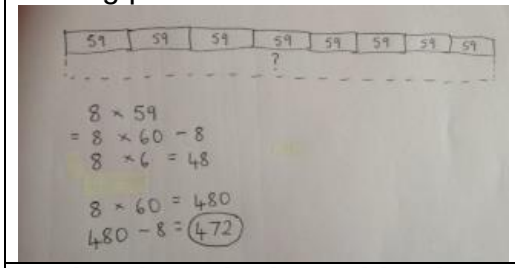
Year 3: solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which n objects are connected to m objects.

Peter has 4 books
Harry has five times as many books as Peter.
How many books has Harry?

4 × 5 = 20
Harry has 20 books

This is an example of integer scaling.

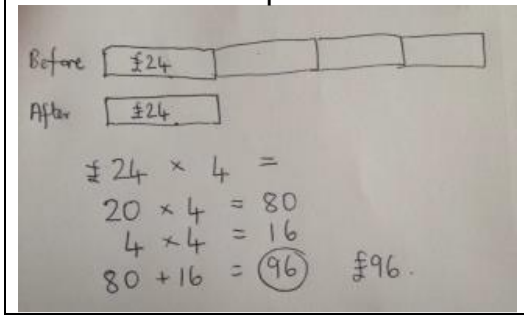
Year 4: solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.



e.g. 8 children each download 59 songs to play on their iPod. How many songs do they have altogether?

National Curriculum Objectives: Fractions

Year 3: solve problems that involve (unit fractions and fractions with the same denominator)
Year 4: solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
Year 4: solve simple measure and money problems involving fractions and decimals to two decimal places, including decimals



e.g. A computer game is £24 in the sale. This is one quarter of its original price. How much did it cost before the sale?

Years 5 and 6:

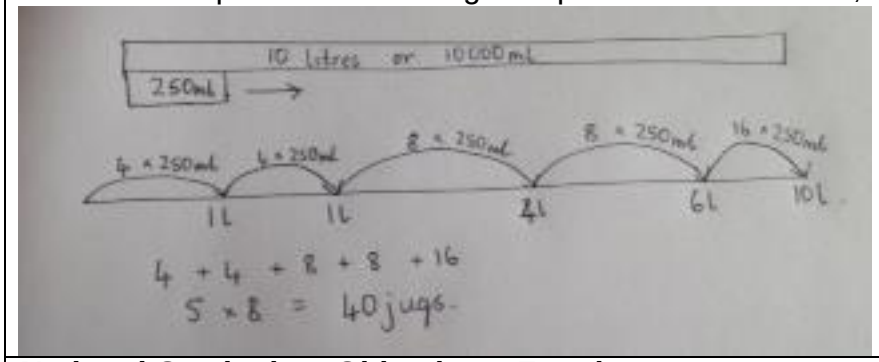
We feel that Cuisenaire and double-sided counters are still valuable resources to use alongside drawn representations of problems. They should all be used flexibly according to the stage that children have reached in their learning.

National Curriculum Objectives: Problem Solving in Addition and Subtraction

Year 5: solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
Year 6: solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

National Curriculum Objectives: Problem Solving in Multiplication and Division

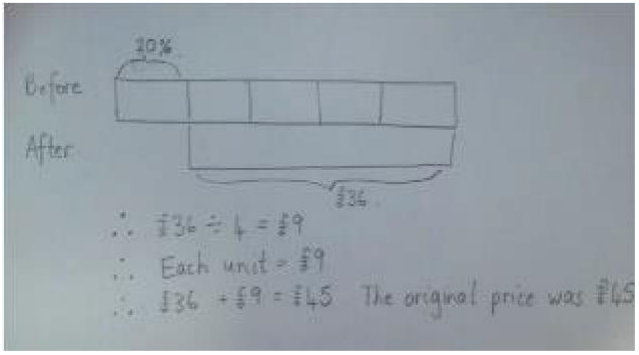
Year 5: solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates



e.g. How many jugs with a capacity of 250ml could you fill with 10 litres of water?

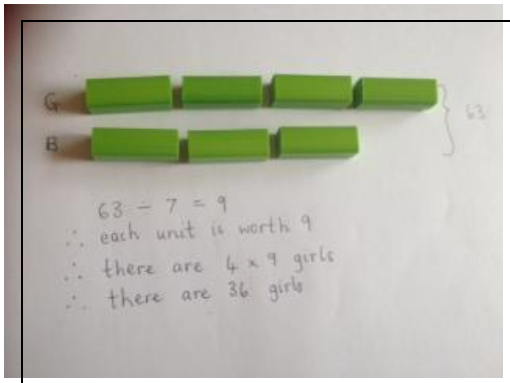
National Curriculum Objectives: Fractions

Year 5: solve problems which require knowing percentage and decimal equivalents of $^{1}/_{2}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{2}/_{5}$, $^{4}/_{5}$ and those with a denominator of a multiple of 10 or 25, including decimals and percentages
Year 6: solve problems which require answers to be rounded to specified degrees of accuracy, including decimals and percentages



e.g. There is 20% off in a sale. The reduced price of the jeans is £36. What was the original price?

The bar model can also be linked beautifully with ratio and proportion, for example, in the following context:



e.g. At a dance there are 4 girls to every 3 boys. There are 63 children altogether? How many girls are there?

With thanks to Kelly DeSantis (BBO Hub) at Longford Primary School for many of the representations above.

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